Supersymmetry without R-Parity and without Lepton Number

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We investigate Supersymmetric models where neither R parity nor lepton number is imposed. Neutrino masses can be kept highly suppressed compared to the electroweak scale if the μ-terms in the superpotential are aligned with the SUSY-breaking bilinear B-terms. This situation arises naturally in the framework of horizontal symmetries. The same symmetries suppress the trilinear R parity violating terms in the superpotential to an acceptable level.

5/95

1. Introduction

Baryon and lepton number conservation are relics of the ancient history of particle physics. We know today that they are not likely to be exactly preserved symmetries of nature. Nonetheless much of the modern discussion of Supersymmetric models is cast within a framework in which symmetries that guarantee baryon and lepton number conservation at the level of renormalizable interactions are assumed [1] [2] [3] [4] [5]. The purpose of the present paper is to show that a much larger spectrum of models may be consistent with the data. Furthermore, since we suspect that the intricate structure of the quark mass matrix is probably connected to a horizontal symmetry group, we find it natural to suppose that this same symmetry group may have something to do with the absence of what are usually called baryon and lepton number violating processes.

Within the Standard Model, lepton number violating observables and lepton flavor changing processes are forbidden because $U(1)_e \times U(1)_\mu \times U(1)_\tau$ is an accidental symmetry of the (renormalizable) Standard Model Lagrangian. This makes such processes particularly sensitive probes of new physics at high energy scales. Thus, measurements of lepton number violating observables such as neutrino (Majorana) masses [6] [7],

$$m_{\nu_e} \leq 5.1 \ eV, \quad m_{\nu_\mu} \leq 160 \ keV, \quad m_{\nu_\tau} \leq 24 \ MeV, \tag{1.1}$$

and lepton flavor changing decays such as [6]

$$BR(\mu \to e\gamma) \leq 4.9 \times 10^{-11}, \quad BR(\mu \to eee) \leq 1.0 \times 10^{-12}, \tag{1.2}$$

put severe constraints on extensions of the Standard Model.

Generic Supersymmetric extensions of the Standard Model predict large contributions to neutrino masses and to lepton flavor violating decays:

(i) Sneutrino VEVs give neutrino masses by mixing neutrinos with the zino $\tilde{z}$ [8] [9] [10] [11].

(ii) Quadratic terms ("μ-terms") in the superpotential give neutrino masses by mixing neutrinos with the (up-)Higgsino $\tilde{\phi}_u^0$ [12] [13].

(iii) Trilinear terms in the superpotential induce tree-level slepton-mediated decays such as $\mu \to 3e$ [4][12][13][14].

1

The bounds (1.1) and (1.2) severely constrain the Supersymmetric parameters. Taking $m_{\tilde{z}} \sim m_Z$, we find

$$\langle \tilde{\nu}_\tau \rangle \lesssim \sqrt{m_{\nu_\tau} m_{\tilde{z}}} \lesssim 1 \; GeV. \tag{1.3}$$

Taking $m_{\tilde{\phi}_u^0} \sim m_Z$, we find

$$\mu_{\nu_\tau \phi_u} \lesssim \sqrt{m_{\nu_\tau} m_{\tilde{\phi}_u^0}} \lesssim 1 \; GeV. \tag{1.4}$$

Both (1.3) and (1.4) become stronger by three orders of magnitude, namely $\langle \tilde{\nu}_\tau \rangle \lesssim 1 \; MeV$ and $\mu_{\nu_\tau \phi_u} \lesssim 1 \; MeV$, if the cosmological bound on long-lived neutrinos, $m(\nu_i) \leq \mathcal{O}(10 \; eV)$, holds. Taking the slepton mass $m_{\tilde{\ell}} \sim m_Z$, the bound (1.2) on $\mu \to 3e$ constrains the product of two lepton number violating couplings to be

$$\lambda_{k12} \lambda_{k11} \lesssim 10^{-6}. \tag{1.5}$$

(We do not consider here the stronger constraints from baryogenesis [15] [16] [17] since they are model dependent [18] and may be evaded in some baryogenesis scenarios [19].)

These bounds pose a serious problem for generic SUSY models where the natural expectation is that $\langle \nu_i \rangle \sim \langle \phi_d \rangle \sim m_Z$, $\mu_{\nu_i \phi_u} \sim \mu_{\phi_d \phi_u} \sim m_Z$ and $\lambda_{ijk} \sim 1$. The standard solution to this problem is to impose a discrete symmetry, R-parity (R_p), that forbids all three types of terms. Alternatively, one could just impose lepton number to forbid these terms.

In this work, we would like to suggest an alternative mechanism to suppress SUSY contributions to neutrino masses: an approximate alignment of the μ terms and the SUSY violating B terms. (Hall and Suzuki [12] noted this case parenthetically in their study of models without R parity but did not emphasize it because it did not fit into the Grand Unified framework which was one of their primary concerns.) We will first present the mechanism and then show that it arises naturally in the framework of abelian horizontal symmetries. Furthermore, such symmetries automatically suppress the trilinear lepton number violating couplings.

2. Alignment

In this section, we introduce our notations, clarify the meaning of the bounds (1.3) and (1.4) and present a mechanism that may satisfy these bounds.

2.1. Notations

In Supersymmetric extensions of the Standard Model without R_p or lepton number, there is a-priori nothing to distinguish the lepton-doublet supermultiplets L_i from the down-Higgs supermultiplet ϕ_d, as both transform as $(2)_{-1/2}$ under $SU(2)_L \times U(1)_Y$. We denote then the four $Y = -1/2$ doublets as L_α, $\alpha = 0, 1, 2, 3$. The single μ-term of the Minimal Supersymmetric Standard Model (MSSM) is now extended to a four-vector,

$$\mu_\alpha L_\alpha \phi_u, \tag{2.1}$$

where $\phi_u(2)_{+1/2}$ is the up-Higgs supermultiplet. The single SUSY breaking B term of the MSSM is also extended to a four-vector,

$$B_\alpha L_\alpha \phi_u, \tag{2.2}$$

where here L_α and ϕ_u stand for the scalar components in the supermultiplets. The trilinear terms in the superpotential contain lepton number violating generalizations of the down quark and charged lepton Yukawa matrices,

$$\lambda_{\alpha\beta k} L_\alpha L_\beta \bar{\ell}_k + \lambda'_{\alpha j k} L_\alpha Q_j \bar{d}_k, \tag{2.3}$$

where $\bar{\ell}_k(1)_{+1}$ are the three lepton singlets, Q_j are quark doublets and $\bar{d}_k$ are down quark singlets. Finally, there are also SUSY breaking scalar masses,

$$m^2_{\alpha\beta} L_\alpha^\dagger L_\beta + \text{h.c.} \tag{2.4}$$

that are relevant to our study. (Here, again, L_α stand for the scalar components.)

2.2. Neutralinos

The full neutralino mass matrix is 7×7, with rows and columns corresponding to $\{\tilde{\gamma}, \tilde{z}, \tilde{\phi}_u^0, \tilde{L}_\alpha^0\}$. (Here, $\tilde{L}_\alpha$ corresponds to the fermionic components in L_α.) Neutrino masses arise from the 6×6 mass matrix M^{n} (the photino is irrelevant to neutrino masses)

$$M^{\mathrm{n}} = \begin{pmatrix} m_{\tilde{z}} & -\frac{g}{2\cos\theta_W} v_u & \frac{g}{2\cos\theta_W} v_\alpha \\ -\frac{g}{2\cos\theta_W} v_u & 0 & \mu_\alpha \\ \frac{g}{2\cos\theta_W} v_\alpha & \mu_\alpha & 0_{4\times 4} \end{pmatrix}, \tag{2.5}$$

3

where $v_u = \langle \phi_u^0 \rangle$, $v_\alpha = \langle L_\alpha^0 \rangle$, and $0_{4\times4}$ denotes a zero 4×4 block in M^n. (The zeros in this 4×4 block are lifted by non-renormalizable terms in the superpotential, *i.e.* $\frac{1}{M}\phi_u\phi_u LL$. Taking $M \gg m_Z$, these terms have negligible effects on our discussion. Here and in our analysis below we neglect these terms as well as additional small loop effects.) In general, M^n gives 4 massive states and two massless ones. Three of the four massive states should correspond to (combinations of) the zino and the two Higgsinos with masses $\sim \mathcal{O}(m_Z)$. The remaining massive state is then one of the neutrinos, and its mass is constrained by (1.1).

The product of the four masses is easily extracted from (2.5). Define

$$\mu \equiv (\sum_\alpha \mu_\alpha^2)^{1/2},$$
$$v_d \equiv (\sum_\alpha v_\alpha^2)^{1/2}, \tag{2.6}$$
$$\cos\xi \equiv \frac{\sum_\alpha v_\alpha \mu_\alpha}{v_d \mu}.$$

Note that ξ measures the alignment of v_α and μ_α. We find

$$\det{}' M^n \sim \mu^2 v_d^2 \sin^2 \xi, \tag{2.7}$$

where by $\det'$ we mean the product of (in our case, the four) eigenvalues different from zero. Following the discussion above, we require

$$\mu^2 v_d^2 \sin^2 \xi \lesssim m_Z^3 m_{\nu_\tau}, \tag{2.8}$$

where m_{ν_τ} stands for the heaviest among the neutrino mass eigenstates: $m_{\nu_\tau} \lesssim 24~MeV$ from direct experiments or $m_{\nu_\tau} \lesssim 10~eV$ from cosmology if its lifetime is longer than the age of the Universe. Note, however, that $\mu \sim \mathcal{O}(m_Z)$ because it provides the charged Higgsino masses, and (for $\tan\beta \sim 1$) $v_d \sim \mathcal{O}(m_Z)$ because it contributes sizably to m_Z and m_W and it provides the down quark and charged lepton masses. The requirement is then

$$\sin\xi \lesssim \mathcal{O}\left(\sqrt{\frac{m_{\nu_\tau}}{m_Z}}\right). \tag{2.9}$$

The bound (2.8) or, equivalently, (2.9) is a severe constraint on SUSY models because generically one expects $\sin\xi \sim \mathcal{O}(1)$. It translates into (1.3) and (1.4) in the following way:

take v_α and μ_α to be approximately aligned. Then there are three mass eigenstates of $M^{\rm n}$ with masses of $\mathcal{O}(m_Z)$. Eq. (2.8) implies that the vev $\langle L_\alpha \rangle$ in the direction orthogonal to these three massive states should be $\lesssim \mathcal{O}(\sqrt{m_Z m_{\nu_\tau}})$ (eq. (1.3)) and, similarly, μ_α in this direction should be $\lesssim \mathcal{O}(\sqrt{m_Z m_{\nu_\tau}})$ (eq. (1.4)).

To summarize, in phenomenologically consistent models, both μ and v_d are of $\mathcal{O}(m_Z)$ and approximately aligned; the misalignment should not exceed $\mathcal{O}(10^{-2})$ or even $\mathcal{O}(10^{-5})$ if the cosmological bound holds.

2.3. Charginos

In the previous sub-section, we have shown that out of the seven neutral fermions, two are (to the approximation in which we work) massless but, in general, five are massive. To guarantee a third very light (compared to the electroweak breaking scale) neutral fermion, v_α and μ_α have to be aligned. There is still a question, however, of whether the three resulting light states correspond to neutrinos. To answer this question, we have to study the charged fermion mass matrix.

The chargino mass matrix $M^{\rm c}$ is 5×5, with rows corresponding to $\{\tilde{w}^-, \tilde{L}_\alpha^-\}$, and columns to $\{\tilde{w}^+, \tilde{\phi}_u^+, \bar{\ell}_k^+\}$:

$$M^{\rm c} = \begin{pmatrix} M_2 & \frac{g}{\sqrt{2}} v_u & 0_{1 \times 3} \\ \frac{g}{\sqrt{2}} v_\alpha & \mu_\alpha & \lambda_{\alpha\beta k} v_\beta \end{pmatrix}. \tag{2.10}$$

Note that the $SU(2)_L$ gauge symmetry implies that $\lambda_{\alpha\beta k}$ is antisymmetric in (α, β) and, therefore, $(M^{\rm c})_{\alpha k} v_\alpha = 0$.

Let us now assume that the phenomenological constraint (2.9) is fulfilled, namely v_α and μ_α are approximately aligned. Then, to a very good approximation, $(M^{\rm c})_{\alpha k} \mu_\alpha = 0$. To understand the consequences, it is convenient to define

$$\phi_d = \frac{1}{v_d} \sum_\alpha v_\alpha L_\alpha, \tag{2.11}$$

and L_i as the three fields orthogonal to ϕ_d. The charged fermion mass matrix with rows corresponding to $\{\tilde{w}^-, \tilde{\phi}_d^-, \tilde{L}_i^-\}$ (and columns as above) is, to a very good approximation, block-diagonal:

$$M^{{\rm c}\prime} = \begin{pmatrix} M_2 & \frac{g}{\sqrt{2}} v_u & 0_{1 \times 3} \\ \frac{g}{\sqrt{2}} v_d & \mu & 0_{1 \times 3} \\ 0_{3 \times 1} & 0_{3 \times 1} & \lambda_{i\phi_d k} v_d \end{pmatrix}. \tag{2.12}$$

(The zeros in the second column stand for highly suppressed entries, of order $\mu \sin \xi$; the other zeros are exact for renormalizable tree-level terms.) We learn the following:

(i) The three singlets $\bar{\ell}_i$ do not mix, to a good approximation, with the triplet $\tilde{w}$ and doublet $\tilde{\phi}_u$. This implies that the mass eigenstates, whose right handed components are $\bar{\ell}_i$, are the 'charged leptons'.

(ii) The left handed components in the charged leptons come from the three L_i.

(iii) Neutrinos, which are defined as the $SU(2)_L$ partners of the left handed charged leptons, correspond then to the three neutral members in L_i.

However, our analysis of the neutralino mass matrix reveals that, for $\mu_\alpha \propto v_\alpha$, the three neutral fermion components in L_i correspond to the three light mass eigenstates. We conclude that aligning μ_α with the VEV v_α guarantees not only that there are three very light neutral fermions, but also that these light states are the three neutrinos.

2.4. Alignment

The alignment of μ_α with v_α,

$$\mu_\alpha \propto v_\alpha, \tag{2.13}$$

can be achieved by imposing two conditions on the SUSY parameters:

(a) The B-terms are proportional to the μ-terms [12]:

$$B_\alpha \propto \mu_\alpha. \tag{2.14}$$

(b) μ_α is an eigenvector of $m^2_{\alpha\beta}$ (the SUSY-breaking scalar mass-squared matrix):

$$m^2_{\alpha\beta}\mu_\beta = \tilde{m}^2\mu_\alpha. \tag{2.15}$$

To prove this statement, note that the minimum equations that determine v_α depend on μ_α, B_α, $m^2_{\alpha\beta}$ and gauge couplings. In particular, the minimum equations do not depend on the trilinear couplings $\lambda_{\alpha\beta k}$ and $\lambda'_{\alpha jk}$, because these always involve a charged field. It is convenient to rotate to a basis where $m^2_{\alpha\beta}$ is diagonal. Condition (b) guarantees that, in this basis, μ_α has only a single component, say μ_0, that is different from zero. Condition (a) guarantees that also B_α has only $B_0 \neq 0$. Then trivially (in similarity to the R parity case) $\langle L_0 \rangle \neq 0$, $\langle L_i \rangle = 0$, is a solution of the minimum equations, namely (2.13) holds.

We conclude that when (2.14) and (2.15) hold, neutrinos do not mix with gauginos and Higgsinos and their masses are, therefore, highly suppressed.

One could think of various theoretical frameworks where (2.14) and (2.15) hold. For example, if string theory guarantees that the μ terms arise from the Kähler potential only and if the quadratic terms in the Kähler potential depend weakly on the moduli whose F-terms break supersymmetry, then B and μ would be approximately aligned. However, in this work we would like to show that the required alignment arises naturally in the framework of horizontal symmetries.

3. Horizontal Symmetries

The hierarchical pattern of fermion masses and mixing angles could be the result of an abelian horizontal symmetry that is explicitly broken by a small parameter. With a single breaking parameter λ, whose charge under the horizontal symmetry is defined to be $H(\lambda) = -1$, the following selection rules apply:

a. Terms in the superpotential that carry charge $n \geq 0$ under $\mathcal{H}$ are suppressed by $\mathcal{O}(\lambda^n)$, while those with $n < 0$ are forbidden due to the holomorphy of the superpotential. (If $\mathcal{H} = Z_N$, the suppression is by $\mathcal{O}(\lambda^{n(\mathrm{mod}N)})$.)

b. Terms in the Kähler potential that carry charge n under $\mathcal{H}$ are suppressed by $\mathcal{O}(\lambda^{|n|})$ (or $\mathcal{O}(\lambda^{\min[\pm n(\mathrm{mod}N)]})$ for $\mathcal{H} = Z_N$).

The selection rules apply to all orders in perturbation theory, so we can safely ignore loop effects.

Note that the μ-terms in the effective low-energy superpotential could originate from either or both of the high energy superpotential and the high energy Kähler potential. The superpotential contributions obey rule a, and their scale is arbitrary. Those from the Kähler potential obey rule b and their natural scale is the SUSY breaking scale $\tilde{m}$ [20] [21].

For the various terms relevant to our study, the following order of magnitude estimates

7

hold (we use $U(1)_Y$ to set $H(\phi_u) = 0$):

$$\mu_\alpha \sim \begin{cases} \mu^0 \lambda^{H(L_\alpha)} & H(L_\alpha) \geq 0, \\ \tilde{m}\lambda^{|H(L_\alpha)|} & H(L_\alpha) < 0, \end{cases}$$

$$B_\alpha \sim \tilde{m}B^0\lambda^{|H(L_\alpha)|}, \qquad (3.1)$$

$$m_{\alpha\beta}^2 \sim \tilde{m}^2\lambda^{|H(L_\beta)-H(L_\alpha)|}.$$

Here, μ^0 and B^0 are unknown 'natural' scales for μ and $B/\tilde{m}$, respectively, and $\tilde{m}$ is the SUSY breaking scale. Eqs. (3.1) lead to the following simple observations:

a. Assuming that all $H(L_\alpha)$ are of the same sign and that one of the L_α fields (say, L_0) carries the smallest horizontal charge, $|H(L_0)| \ll |H(L_i)|$ $(i = 1, 2, 3)$, then both the μ_α terms and the B_α terms will be dominantly in the direction of this field:

$$\mu_0 \gg \mu_i, \quad B_0 \gg B_i. \qquad (3.2)$$

b. The diagonal terms in $m_{\alpha\beta}^2$ are not suppressed by the selection rules, namely $m_{\alpha\alpha}^2 \sim \tilde{m}^2$, while the off-diagonal are suppressed if the various fields have different charges,

$$m_{\alpha\beta}^2 \ll m_{\alpha\alpha}^2 \sim \tilde{m}^2 \quad (\alpha \neq \beta). \qquad (3.3)$$

The important point here is that $\frac{m_{0i}^2}{m_{00}^2} \sim \frac{\mu_i}{\mu_0}$.

These two effects fulfill the two conditions described in the previous section in an *approximate* way. Consequently, the mixings of neutrinos with the zino and the Higgsino do not vanish but are suppressed. It now becomes a *quantitative* question of whether reasonable horizontal charge assignments lead to satisfactory suppression of neutrino masses.

Note that, since the mixing between L_0 and the three L_i is very small, we can neglect the rotation (2.11) from the $\{L_\alpha\}$ basis to the $\{\phi_d, L_i\}$ basis in our various order of magnitude estimates.

The quantitative answer is easy to find: as $\sin\xi \sim \mathcal{O}(\frac{\mu_i}{\mu_0}) \sim \mathcal{O}(\frac{B_i}{B_0}) \sim \mathcal{O}(\frac{m_{0i}^2}{m_{00}^2})$, eq. (2.9) (or, equivalently, (1.3) and (1.4)) is satisfied if

$$\lambda^{H(L_\tau)-H(\phi_d)} \lesssim \sqrt{\frac{m_{\nu_\tau}}{m_Z}} \lesssim \begin{cases} 10^{-2} & m_{\nu_\tau} \leq 24 \ MeV, \\ 10^{-5} & m_{\nu_\tau} \lesssim 10 \ eV. \end{cases} \qquad (3.4)$$

If the small parameter $\lambda \sim 0.2$, as suggested by the magnitude of the Cabibbo angle, then

$$H(L_\tau) - H(\phi_d) \gtrsim \begin{cases} 3 & m_{\nu_\tau} \leq 24 \ MeV, \\ 7 & m_{\nu_\tau} \lesssim 10 \ eV. \end{cases} \qquad (3.5)$$

8

A charge difference of $\mathcal{O}(7)$ may be too large for reasonable models. However, in some models of ref. [22], where the symmetry breaking parameters are much smaller than 0.2, the required approximate alignment can be achieved with charge differences ≤ 2.

Eq. (3.5) ensures that, at tree level, m_{ν_τ} is safely suppressed. One may still worry whether loop corrections can give larger contributions to the neutrino masses. However, this is not the case. The leading contributions come from loops generated by the λ'_{ijk} couplings (2.3) with d-type quarks–squarks circulating in the loop. They are proportional to the heaviest d-quark mass m_b : $(\delta m_\nu)_{ij} \sim \frac{1}{16\pi^2}\lambda'_{i3k}\lambda'_{jk3}m_b$. Since $\lambda'_{i3k} \lesssim \lambda^{H(L_i)-H(\phi_d)}\frac{m_b}{m_Z}$, the radiative contributions to m_{ν_i} require

$$\lambda^{H(L_i)-H(\phi_d)} \lesssim \sqrt{\frac{m_{\nu_i}}{m_Z}}\,4\pi\,\left(\frac{m_Z}{m_b}\right)^{3/2}. \tag{3.6}$$

This is weaker than (3.4) by a factor $\sim 10^3$. Eq. (3.6) shows explicitly how the suppression from horizontal symmetries is effective at any order in perturbation theory, and indeed justifies neglecting loop effects.

We conclude that in models of abelian horizontal symmetries, the μ and B terms are dominantly in the direction of one of the four L_α fields, and the scalar mass–squared matrix does not significantly mix this field with the other three. This leads to an approximate alignment of μ_α and v_α. Consequently, neutrino masses from mixing with the zino or Higgsino can be suppressed well below the electroweak scale, while radiative contributions can be kept negligibly small. Whether this suppression is strong enough is a model dependent question. We present a class of models with satisfactory suppression in the next section.

4. An Explicit Example

Take a model with an exact discrete horizontal symmetry

$$\mathcal{H} = Z_{n_1} \times Z_{n_2}. \tag{4.1}$$

The symmetry is spontaneously broken – as we show below – by two scalars in singlet supermultiplets:

$$S_1(-1,0), \quad S_2(0,-1), \tag{4.2}$$

In addition, we have the doublet supermultiplets:

$$\phi_u(0,0), \quad L_\alpha(H_{1\alpha}, H_{2\alpha}). \tag{4.3}$$

We use horizontal charges $0 \le H_{i\alpha} \le n_i - 1$.

In order to estimate the VEVs of the various fields, we investigate the Higgs potential and the minimum equations. We assume that there are only two scales in the model: $\tilde{m}$ is the SUSY breaking scale which characterizes all SUSY breaking terms, and M_p, the Planck scale which suppresses all non-renormalizable terms. We consider all the terms that are consistent with $SU(2)_L \times U(1)_Y \times \mathcal{H}$. We omit dimensionless coefficients of $\mathcal{O}(1)$ in all formulae.

The leading terms in the superpotential are

$$W \sim \frac{S_1^{n_1}}{M_p^{n_1-3}} + \frac{S_2^{n_2}}{M_p^{n_2-3}} + \sum_\alpha \frac{S_1^{H_{1\alpha}} S_2^{H_{2\alpha}} (\phi_u L_\alpha)}{M_p^{H_{1\alpha}+H_{2\alpha}-1}}. \tag{4.4}$$

They lead to the following (leading) terms in the Higgs potential:

$$\begin{aligned} V^W \sim & \frac{|S_1|^{2n_1-2}}{M_p^{2n_1-6}} + \frac{|S_2|^{2n_2-2}}{M_p^{2n_2-6}} \\ & + \sum_\alpha \left[\left(\frac{S_1^*}{M_p}\right)^{n_1-1} \frac{S_1^{H_{1\alpha}-1} S_2^{H_{2\alpha}}}{M_p^{H_{1\alpha}+H_{2\alpha}-3}} + \left(\frac{S_2^*}{M_p}\right)^{n_2-1} \frac{S_1^{H_{1\alpha}} S_2^{H_{2\alpha}-1}}{M_p^{H_{1\alpha}+H_{2\alpha}-3}} \right] (\phi_u L_\alpha) + \text{h.c.}. \end{aligned} \tag{4.5}$$

In addition, there are D terms in the Higgs potential,

$$V^D \sim (|\phi_u|^2 - \sum_\alpha |L_\alpha|^2)^2, \tag{4.6}$$

soft scalar masses (off-diagonal terms are highly suppressed),

$$V^A \sim \tilde{m}^2 (|S_1|^2 + |S_2|^2 + |\phi_u|^2 + \sum_\alpha |L_\alpha|^2), \tag{4.7}$$

and (the leading) soft SUSY breaking terms analytic in the fields,

$$V^B \sim \tilde{m} \left[\frac{S_1^{n_1}}{M_p^{n_1-3}} + \frac{S_2^{n_2}}{M_p^{n_2-3}} + \sum_\alpha \frac{S_1^{H_{1\alpha}} S_2^{H_{2\alpha}} (\phi_u L_\alpha)}{M_p^{H_{1\alpha}+H_{2\alpha}-1}} \right]. \tag{4.8}$$

In (4.5), (4.6), (4.7) and (4.8), the various fields stand for the neutral scalar components.

Solving the minimum equations for $\langle S_i \rangle$, we get the two small breaking parameters for H:

$$\lambda_1 \equiv \frac{\langle S_1 \rangle}{M_p} \sim \left(\frac{\tilde{m}}{M_p} \right)^{\frac{1}{n_1 - 2}}, \quad \lambda_2 \equiv \frac{\langle S_2 \rangle}{M_p} \sim \left(\frac{\tilde{m}}{M_p} \right)^{\frac{1}{n_2 - 2}}. \tag{4.9}$$

This is a generalization of the $\mathcal{H} = Z_n$ case studied in ref. [23]. For the scalar doublet VEVs, we get

$$\frac{\langle \phi_u \rangle}{\tilde{m}} \sim 1, \tag{4.10}$$

$$\frac{\langle L_\alpha \rangle}{\tilde{m}} \sim \left(\frac{\tilde{m}}{M_p} \right)^{\frac{H_{1\alpha}}{n_1 - 2} + \frac{H_{2\alpha}}{n_2 - 2} - 1}. \tag{4.11}$$

Equation (4.11) shows that, indeed, the VEVs of the L_α doublets are hierarchical and depend on the horizontal charges in the way described in the previous section. The effective μ_α can be extracted from eq. (4.4) by putting in the VEVs $\langle S_i \rangle$, with the result $\mu_\alpha^{\text{eff}} \sim \langle L_\alpha \rangle$. That is, μ_α and $\langle L_\alpha \rangle$ are approximately aligned. Taking $\langle L_0 \rangle > \langle L_3 \rangle$ to be the two largest of the four $\langle L_\alpha \rangle$, the alignment is accurate to order $\lambda_1^{H_1(L_3) - H_1(L_0)} \lambda_2^{H_2(L_3) - H_2(L_0)}$.

The VEV of the down Higgs (for $\tan\beta \sim 1$) should be of $\mathcal{O}(\tilde{m})$. This is achieved if one of the L_α fields, say L_0, has one of its horizontal charges $H_i(L_0) = n_i - 2$ and the other $H_j(L_0) = 0$:

$$L_0(n_1 - 2, 0) \implies \langle L_0 \rangle, \mu_0^{\text{eff}} \sim \tilde{m}. \tag{4.12}$$

If we then take, for example,

$$L_i(H_{1i}, n_2 - 2) \implies \langle L_i \rangle, \mu_i \sim \tilde{m} \left(\frac{\tilde{m}}{M_p} \right)^{\frac{H_{1i}}{n_1 - 2}}, \tag{4.13}$$

so that

$$\frac{H_{1i}}{n_1 - 2} \gtrsim \frac{1}{3} \implies \langle L_i \rangle, \mu_i \lesssim 10^{-5} \tilde{m}, \tag{4.14}$$

the alignment is precise to $\mathcal{O}(10^{-5})$, and neutrino masses are safely below the cosmological bound.

The model presented here, in addition to naturally suppressing neutrino masses, has two more attractive features [23]:

1. Eq. (4.9) shows that a hierarchy of VEVs that could be relevant to fermion parameters can arise naturally out of the initial two-scale model.

2. Eq. (4.12) shows that the horizontal symmetry can naturally solve the μ-problem.

The model may seem complicated, but the reason is that we want to demonstrate the power of horizontal symmetries in naturally achieving these extra advantages. A model with a gauged horizontal $U(1)$ symmetry, with given small breaking parameters and a given scale μ^0, would achieve the required alignment in lepton parameters with much simpler charge assignments. (See, for example, the models of ref. [24].)

We also note that a model without R_p and with the L_α transforming non-trivially under a single horizontal Z_n does not work. The VEVs of the doublet fields are $\langle L_\alpha \rangle \sim \tilde{m} \left(\frac{\tilde{m}}{M_p} \right)^{\frac{H_\alpha}{n-2}-1}$. Consequently, for $H_\alpha < n - 2$ (which is unavoidable for some of the horizontal charges) $\langle L_\alpha \rangle > \tilde{m}$, and the electroweak symmetry is broken at a scale higher than the SUSY breaking scale. Of course, with R_p, models with a single Z_n and $H(\phi_d) = n - 2$ do solve the μ problem [23].

To demonstrate the full power of the discrete horizontal symmetry, we suggest the following explicit example. Take $\mathcal{H} = Z_{14} \times Z_{10}$, with S_i of eq. (4.2), ϕ_u of eq. (4.3), and

$$L_0(12, 0), \ L_3(4, 8), \tag{4.15}$$

(and higher charges for L_1, L_2). Solving the minimum equations and studying the neutrino spectrum, we find:

(i) The two small breaking parameters are

$$\lambda_1 \equiv \frac{\langle S_1 \rangle}{M_p} \sim \lambda^2, \quad \lambda_2 \equiv \frac{\langle S_2 \rangle}{M_p} \sim \lambda^3, \tag{4.16}$$

which could explain all quark and lepton parameters, as shown in refs. [22][23][24].

(ii) The μ-problem is solved namely, identifying $\phi_d \sim L_0$,

$$\langle \phi_u \rangle \sim \langle \phi_d \rangle \sim \mu_0 \sim \tilde{m}. \tag{4.17}$$

(iii) Neutrino masses are highly suppressed. In particular, $m_{\nu_\tau} \sim 10 \ eV$ which is the relevant range for being hot dark matter.

5. Trilinear Lepton Number Violating Terms

We investigate the dimension-4 terms in the superpotential:

$$\lambda_{ijk} L_i L_j \bar{\ell}_k + \lambda'_{ijk} L_i Q_j \bar{d}_k + \lambda''_{ijk} \bar{u}_i \bar{d}_j \bar{d}_k. \tag{5.1}$$

(Non-renormalizable lepton number violating terms pose no problem.) In our presentation below we neglect the rotation from the interaction basis (where the horizontal charges are well defined) to the mass basis: a full analysis, involving an estimate of the mixing angles (which are also determined by the horizontal charges), would give just the same order of magnitude estimates.

The selection rules, when applied to these couplings, imply

$$\lambda_{ijk} \sim \begin{cases} \lambda^{H(L_i)+H(L_j)+H(\bar{\ell}_k)} & H(L_i) + H(L_j) + H(\bar{\ell}_k) \geq 0 \\ 0 & H(L_i) + H(L_j) + H(\bar{\ell}_k) < 0 \end{cases}$$
$$\lambda'_{ijk} \sim \begin{cases} \lambda^{H(L_i)+H(Q_j)+H(\bar{d}_k)} & H(L_i) + H(Q_j) + H(\bar{d}_k) \geq 0 \\ 0 & H(L_i) + H(Q_j) + H(\bar{d}_k) < 0 \end{cases}. \tag{5.2}$$

We first assume baryon number conservation (which requires $\lambda''_{ijk} = 0$). Stringent bounds apply to products of two λ's [4][25] [26] [27] [28]. These are given in Table I.

Table I: Constraints on Lepton Number Violating Couplings

Couplings	Limit	Process	Master model
$\lambda'_{k12}\lambda'_{k21}$	$9 \times 10^{-8} \sim \lambda^{10}$	Δm_K	λ^{10}
$\lambda'_{k12}\lambda'_{k21}$	$8 \times 10^{-10} \sim \lambda^{13}$	ϵ	λ^{10}
$\lambda'_{k13}\lambda'_{k31}$	$4 \times 10^{-6} \sim \lambda^{8}$	Δm_B	λ^{8}
$\lambda_{k12}\lambda_{k11}$	$10^{-4} \sim \lambda^{6}$	$\mu \to eee$	λ^{15}
$\lambda_{2jk}\lambda_{1jk}$	$10^{-3} \sim \lambda^{5}$	$\mu \to e\gamma$	λ^{12}
$\lambda'_{k21}\lambda_{k21}$	$2 \times 10^{-5} \sim \lambda^{7}$	$K_L \to \mu e$	λ^{11}
$\lambda'_{k21}\lambda_{k22}$	$10^{-4} \sim \lambda^{6}$	$K_L \to \mu\mu$	λ^{10}
$\lambda'_{k21}\lambda_{k11}$	$2 \times 10^{-5} \sim \lambda^{7}$	$K_L \to ee$	λ^{13}

Note the following:

(a) All bounds correspond to $\tilde{m} = 1\, TeV$ and scale like $1/\tilde{m}^2$.

(b) The bounds from K_L decays also apply to $\lambda_{k21} \to \lambda_{k12}$, $\lambda'_{k21} \to \lambda'_{k12}$. In all these other cases the horizontal symmetry gives similar or even stronger suppression.

(c) We do not present various additional bounds that require λ_{ijk}, $\lambda'_{ijk} \lesssim \lambda^2$ and are easily satisfied in any of our models.

13

(d) The master model for quarks was presented in ref. [23]. For the lepton sector, we assume [24]

$$V_{e\mu} \sim \lambda^2, \quad \frac{m_{\nu_\mu}}{m_{\nu_\tau}} \sim \lambda^4, \quad \frac{m_e}{m_\mu} \sim \lambda^3, \quad \frac{m_\mu}{m_\tau} \sim \lambda^2. \tag{5.3}$$

The conclusion is that all the bounds are satisfied within our horizontal symmetry models. The only potential problem is in ϵ if we assume phases of $\mathcal{O}(1)$. This can be solved by a slight modification of the 'master' model: Choosing horizontal charges $H' = H + \alpha L$ (where L is lepton number and α is a real coefficient), one can achieve an arbitrary suppression of the lepton number violating terms in (5.1), while the only effect on the fermion mass matrices is an overall suppression of all neutrino masses.

Baryon number violation was investigated in ref. [29] assuming massless neutrinos. With slight modifications of their models, a satisfactory suppression of proton decay can be achieved for the massive neutrino case as well.

To summarize: Assuming baryon number conservation, dimension 4 lepton number violating terms are suppressed to a phenomenologically acceptable level by a horizontal symmetry. In ref. [29] models were constructed in which horizontal symmetry rather than baryon number suppresses proton decay and other B violating processes. Simple modifications of those models lead to a horizontal symmetry framework in which all of the usual phenomenological consequences of baryon number, lepton number and R parity follow.

6. Conclusions

Supersymmetric models without R parity and without lepton number symmetry lead, in general, to an unacceptably large neutrino mass. This problem is solved, however, in any model where (similarly to models with R parity), the vacuum expectation value of the four $Y = -1/2$ doublet scalars is aligned with the μ term which couples these fields to the $Y = +1/2$ doublet scalar. For this alignment to arise, two conditions have to hold: the soft SUSY breaking B term is proportional to the μ term, and the μ term is an eigenvector of the SUSY breaking scalar masses of the $Y = -1/2$ doublet scalars.

Models of abelian horizontal symmetries, with charges dictated by fermion masses and mixing, automatically fulfill these conditions but in an approximate way. The resulting approximate alignment could lead to satisfactorily small neutrino masses. In addition, trilinear lepton number violating terms in the superpotential are allowed but suppressed below experimental constraints. The resulting phenomenology could differ significantly from models with exactly conserved R parity in low energy processes [14][28][27][30] [31], in collider experiments [25][26][32] [33] [34] [35] [36], in the cosmological consequences [37] [38] [39] and in some more peculiar effects, *e.g.* matter enhanced neutrino oscillations [40] [41] [42]. Most prominently, in the present framework, there is no reason for the existence of a stable LSP. This will change both the cosmological and laboratory signals for supersymmetry.

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